Name: _____

Instructor: _____

Math 10560, Practice Exam 3
April 24, 2023

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
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9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

Total _____

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Multiple Choice

1.(7 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(a) converges absolutely.

(b) diverges because the terms alternate.

(c) diverges even though $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$.

(d) diverges because $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$.

(e) does not converge absolutely but does converge conditionally.

2.(7 pts.) Use Comparison Tests to determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$

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3.(7 pts.) Consider the following series

$$(I)\quad \sum_{n=1}^{\infty}\left(\frac{2n^2+7}{n^2+1}\right)^n \qquad (II)\quad \sum_{n=2}^{\infty}\frac{2^{1/n}}{n-1} \qquad (III)\quad \sum_{n=1}^{\infty}\frac{n!}{e^n}$$

Which of the following statements is true?

(a) (I) converges, (II) diverges, and (III) converges.

(b) They all converge.

(c) They all diverge.

(d) (I) diverges, (II) diverges, and (III) converges.

(e) (I) converges, (II) diverges, and (III) diverges.

4.(7 pts.) Which series below conditionally converges?

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n^3}}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}e^n}{\sqrt{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$ (e) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}$

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5.(7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\dfrac{x^2}{1+x}$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \; x^{n+2}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} x^{2n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$

6.(7 pts.) Which series below is a power series for $\cos(\sqrt{x})$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{\,n}}{(2n)!}$

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7.(7 pts.) Calculate

$$\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9}.$$

Hint: Without MacLaurin series this may be a long problem.

(a) 0 (b) $\frac{9}{7}$ (c) $\frac{7}{9}$ (d) ∞ (e) $-\frac{1}{6}$

8.(7 pts.) Find a power series representation for the the function $f(x) = \ln(1 - x^2)$.
Hint: $\frac{d}{dx}\ln(1 - x^2) = \frac{-2x}{1 - x^2}$.

(a) $\sum_{n=0}^{\infty}(-2)(2n+1)x^{2n}$ (b) $\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n+2}}{2n+2}$ (c) $\sum_{n=0}^{\infty}\frac{(-2)x^{2n+2}}{2n+2}$

(d) $\sum_{n=0}^{\infty}(-2)^n x^{2n}$ (e) $\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n+1}}{2n+1}$

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9.(7 pts.) What is the fourth Taylor polynomial, $T_4(x)$, for $\cos(2x)$ with center $a = \pi$?

(a) $1 - 4(x-\pi)^2 + 16(x-\pi)^4$

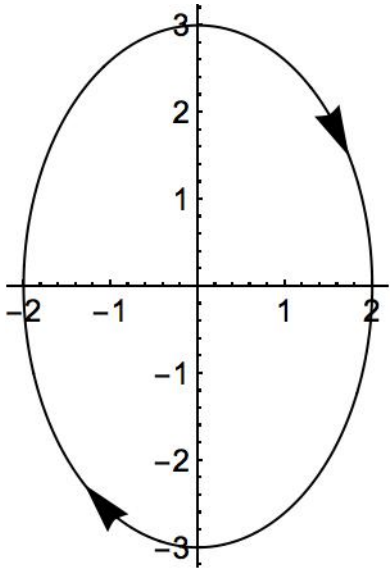
(b) $1 - \frac{1}{2!}(x-\pi)^2 + \frac{1}{4!}(x-\pi)^4$

(c) $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$

(d) $1 - 2(x-\pi)^2 + \frac{2}{3}(x-\pi)^4$

(e) $1 + 4(x-\pi)^2 + 16(x-\pi)^4$

10.(7 pts.)



The graph of the parametric curve shown above is the graph of which of the following parametric equations?

(a) $x(t) = 3\cos(t), \quad y(t) = 2\sin(t), \quad 0 \le t \le 2\pi.$

(b) $x(t) = 2\cos(t), \quad y(t) = 3\sin(t), \quad 0 \le t \le 2\pi.$

(c) $x(t) = 2\sin(t), \quad y(t) = 3\cos(t), \quad 0 \le t \le 2\pi.$

(d) $x(t) = \frac{3}{2}\sin(t), \quad y(t) = \cos(t), \quad 0 \le t \le 2\pi.$

(e) $x(t) = 3\sin(t), \quad y(t) = 2\cos(t), \quad 0 \le t \le 2\pi.$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(11 pts.) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}}$$

converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

Remark: The correct answer with no justification is worth 2 points.

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12.(11 pts.) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x-3)^n$$

Remark: The correct answer with no justification is worth 2 points.

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13.(11 pts.)

(a) Show that

$$\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that $|x| < 1$.

(b) Find

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

(**Hint:** First use term-by-term integration on the series in part (a).)

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